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The effect of heat generation on convective heat transfer for laminar flows in a multi-passage circular pipe maintained under constant wall temperature

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INTRODUCTION

A LITERATURE survey [1–10] indicates that heat transfer studies for laminar flows in multi-passage circular pipes based on temperature distributions including heat generation effects are not available. The existing conventional convective heat transfer problems in flows of multi-passage tubes are based on lumped parameter type analyses [1] which are not rigorous. However, recent interest in the heat generation effect on convective heat transfer properties [11] has led to this note where the effect of an arbitrary heat generation density in the inner and outer flows on convective heat transfer properties of the multi-passage flow is studied. The thickness of the separating surface and the resistance of heat conductivity of the inner separating wall are neglected. It has been shown that the Nusselt numbers on the outer surface and at the inner separating surface of a multi-passage pipe flow can be defined in three different ways. Also shown was that the Nusselt numbers depend on two dimensionless parameters [12]: the ratio of the thermal conductivities of the fluids in the inner and outer passages, k_k, and the product of k_k and the ratio of the Peclet numbers of the flows in the inner and outer passages, $\eta = k_k k_{Pe}$, the heat exchanger number of the multi-passage flow.

The effect of heat generation is reflected by the dimensionless heat generation numbers γ_i and γ_o for the inner and outer flows, respectively, where $\gamma = (LH)/[kC(Pe)]$, L is the radius of the conduit, H the heat generation volume density, k the thermal conductivity, and Pe the Peclet number [11, 13]. The Nusselt numbers in this multi-passage flow are defined relative to the mixed, combined bulk temperature of the inner and outer region flows.

TEMPERATURE FIELDS

By neglecting the thermal resistance of the inner separation wall, and considering the sections located sufficiently removed from both entrances of the conduit where hydrodynamically and thermally fully developed flow conditions prevail, the continuity of the temperature distribution is maintained by the forms:

$$T_o = CZ+E_o(X,Y),\quad T_i = CZ+E_{in}+E_i(X,Y) \tag{1}$$

where T_o, T_i and E_o, E_i indicate the temperature and excess temperature distributions in the outer and inner regions, respectively, for a constant heat flux condition, and E_{in} represents the temperature at the inner separation surface for a constant heat flux case.

The continuity of the two temperature fields to be determined is satisfied by

$$\bar{T}_o = \bar{E}_o(X,Y) \quad\text{and}\quad \bar{T}_i = \bar{E}_{in}+\bar{E}_i \tag{2}$$

together with the boundary condition $\bar{T}_o = 0$ at the wall.

It must be noted that this selection cannot impose any restriction and it can be used without any loss of generality of the problem. Therefore, the quantities $\bar{T}_o$, $\bar{T}_i$, $\bar{E}_o$, $\bar{E}_i$ and $\bar{E}_{in}$ represent temperatures and cap temperatures in the outer, and inner regions, and at the separation surface, respectively, for the case of constant wall temperature.

The energy equations to be satisfied in the inner and outer flow regions are

$$\begin{aligned}\frac{CZ-T_o}{CZ-T_m}CW_o &= \alpha\left(\frac{\partial^2\bar{T}_o}{\partial X^2}+\frac{\partial^2\bar{T}_o}{\partial Y^2}+\frac{H_o}{k_o}\right)\\ \frac{CZ-T_i}{CZ-T_m}CW_i &= \alpha\left(\frac{\partial^2\bar{T}_i}{\partial X^2}+\frac{\partial^2\bar{T}_i}{\partial Y^2}+\frac{H_i}{k_i}\right)\end{aligned} \tag{3}$$

where T_m is the mixed mean temperature of the combined flow [12].

The energy equations to be satisfied, suitable for the constant wall temperature condition, in each flow region expressed in dimensionless variables are [13]

$$ac(Pr)_o e_o w_o - h_o = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{e}_o}{\partial r}\right) \quad \text{for } \omega < r < 1 \tag{4}$$

$$ac(Pr)_i(e_i+e_{in})w_i - \frac{k_o}{k_i}h_i = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{e}_i}{\partial r}\right) \quad \text{for } 0 < r < \omega \tag{5}$$

where

$$a = \frac{1+k_c\lambda}{e_{mo}+k_c\lambda e_{in}+k_c\lambda e_{mi}} \tag{6}$$

and r is the dimensionless radial coordinate defined relative to the conduit radius L.

The boundary conditions for $\bar{e}_o$ and $\bar{e}_i$ are

$$\begin{aligned}&\bar{e}_o = 0,\quad \frac{d\bar{e}_i}{dr} = 0 \quad \text{for } r = 0\\ &\bar{e}_o = \bar{e}_{in},\quad \bar{e}_i = 0 \quad \text{for } r = \omega\end{aligned} \tag{7}$$

where the functions e_o and e_i are given in ref. [12].

The solutions to equations (4) and (5) under the boundary conditions shown in equations (7) are determined to be

$$\begin{aligned}\bar{e}_o = aG_o\Big\{&\frac{e_{in}}{\ln\omega}(B_1\ln^2 r+B_2r^2\ln r+B_3\ln r\\ &+B_4r^2+B_5)r^2+G_o[B_6r^2\ln^2 r+B_7\ln^2 r\\ &+B_8r^4\ln r+B_9r^2\ln r+B_{10}\ln r+B_{11}r^6\\ &+B_{12}r^4+B_{13}r^2+B_{14}]r^2+M_1\ln r+N_1\Big\}\end{aligned} \tag{8}$$

$$\bar{e}_i = aG_i\left[\frac{1}{16}\frac{e_{in}}{\omega^3}(4\omega^2-r^2)r^2 - \frac{1}{16}\frac{G_i}{\omega^6}\left(\frac{3}{4}\omega^6 - \frac{7}{16}\omega^4 r^2 + \frac{5}{36}\omega^2 r^4 - \frac{1}{64}r^6\right)r^2 + \frac{1}{4}\frac{\gamma_i G_o}{k_k}\left(\frac{1}{36}\frac{r^4}{\omega^3} - \frac{1}{8}\frac{r^2}{\omega} + \frac{1}{4}\omega - \frac{1}{aG_i}\right)r^2 + M_2 \ln r + N_2\right] \quad (9)$$

where the B's, M_1, N_1, M_2, N_2 are calculated explicitly in terms of ω, γ_o, γ_i.

Heat transfer continuity on the inner separating surface is satisfied by the relation

$$k_o\left(\frac{\partial \bar{e}_o}{\partial r}\right)_{r=\omega} = k_i\left(\frac{\partial \bar{e}_i}{\partial r}\right)_{r=\omega}. \quad (10)$$

This condition, after introducing an alternate dimensionless temperature on the separating surface as $\beta = \bar{e}_{in}/G_o$ will yield

$$\beta = -aG_o\omega\left[\beta\left(C_1 \ln^2\omega + C_2\ln\omega + C_3\frac{1}{\ln\omega} + C_4\right) + C_5\ln^3\omega + C_6\ln^2\omega + C_7\ln\omega + C_8 - \frac{1}{16}\left(4\beta\omega^3 - \frac{\eta}{k_k}C_9\right)\eta\ln\omega - \frac{1}{2}\left(\frac{1}{12}\frac{G_i}{G_o}\omega - \frac{1}{aG_o}\right)\gamma_i\omega\ln\omega\right]. \quad (11)$$

The C's involved in β are calculated explicitly in terms of ω, γ_o, γ_i.

HEAT FLUXES AND HEAT TRANSFER COEFFICIENTS

The rate of heat flow per unit length of conduit through the outer and inner surfaces, considered positive when flowing into the outer region, is expressed, respectively, as [14]

$$U_o = LF2\pi\left(\frac{\partial \bar{e}_o}{\partial r}\right)_{r=1}, \quad U_i = LF2\pi\omega\left(\frac{\partial \bar{e}_o}{\partial r}\right)_{r=\omega}. \quad (12)$$

Substituting $\bar{e}_o$ and $\bar{e}_i$ from equations (8) and (9), respectively, into equations (12)

$$U_o = 2\pi LFG_o\left[aG_o\left(\frac{\beta}{\ln\omega}D_1 + D_2\right) + \frac{\beta}{\ln\omega}\right]$$
$$U_i = -2\pi LF\omega G_o\left[aG_o\left(\frac{\beta}{\ln\omega}E_1 + E_2\right) + \frac{\beta}{\omega\ln\omega}\right] \quad (13)$$

where factors D and E are calculated explicitly in terms of ω, γ_o, γ_i.

The Nusselt numbers are defined in this paper according to the definition given in ref. [12], for the case where the inner and outer wall heat transfer coefficients are based on the mixed flow bulk temperature of the combined inner and outer flows. The mixed flow bulk temperature is expressed as

$$\bar{T}_m \frac{1}{C_{po}Q_o + C_{pi}Q_i}\left(C_{po}\rho_o\int_{A_o} W_o\bar{T}_o\,dA + C_{pi}\rho_i\int_{A_i} W_i\bar{T}_i\,dA\right) \quad (14)$$

where C_{po}, A_o and C_{pi}, A_i are the specific heat and cross-sectional area for the outer and inner passage, respectively.

The heat transfer coefficients on the outer wall and on the inner separating surface ($\bar{h}_o$, $\bar{h}_i$) are calculated from

$$U_o = (\bar{T}_{wall} - \bar{T}_m)2\pi L\bar{h}_o, \quad U_i = (\bar{T}_{wall} + \bar{E}_{in} - \bar{T}_m)2\pi\omega L\bar{h}_i \quad (15)$$

where $\bar{T}_{wall} = 0$.

Selecting the fluid in the outer passage as the reference fluid, the Nusselt number on each surface based on its respective diameter is defined by

$$(Nu)_o = 2L\bar{h}_o/k_o, \quad (Nu)_i = 2\omega L\bar{h}_i/k_o. \quad (16)$$

The expression for a in equation (6) can be changed into

$$aG_o = \frac{I_{oo} + \eta\omega^4/4}{-J_o/G_o + \eta\omega^4\beta/4 - \eta J_i/G_o}. \quad (17)$$

Similarly, the factor aG_i in equation (9) can be expressed as

$$aG_i = -aG_o\eta\omega^3/k_k. \quad (18)$$

Equating the heat flow expressions in equations (13) and (15) and simplifying gives

$$aG_o\left(D_1\frac{\beta}{\ln\omega} + D_2\right) + \frac{\beta}{\ln\omega} = \frac{1}{2}\frac{\frac{\bar{J}_o}{G_o} + \eta\frac{\bar{J}_i}{G_o} - \frac{1}{4}\eta\omega^4\beta}{I_{oo} + \frac{1}{4}\eta\omega^4}(Nu)_o$$
$$aG_o\left(E_1\frac{\beta}{\ln\omega} + E_2\right)\omega + \frac{\beta}{\ln\omega} = -\frac{1}{2}\frac{\frac{\bar{J}_o}{G_o} + \eta\frac{\bar{J}_i}{G_o} + \beta I_{oo}}{I_{oo} + \frac{1}{4}\eta\omega^4}(Nu)_i. \quad (19)$$

DISCUSSION OF NUMERICAL RESULTS AND CONCLUSIONS

A heat generation number $\gamma = 30.0$ for both inner and outer region flows and a numerical value for the ratio of the thermal conductivities of the two fluids $k_k = 1$ are selected as an example for numerical calculations. Using four fixed values for the dimensionless heat exchanger number η (1, 10, -1, -10), curves are plotted in ratio forms in order to see the numerical influence of heat generation on the Nusselt numbers, as the ratios of the Nusselt numbers without to with heat generation; Figs. 1 and 2, respectively. The numerical value of the heat generation number used, was calculated from the data given in ref. [15]. This value corresponds to practical applications including various radioactive flow conditions.

In Fig. 1 a comparison of the Nusselt numbers without heat generation to that with heat generation is presented by plotting the ratio of (Nu_o) without heat generation to (Nu_o) with heat generation, against the dimensionless radius of the separating wall. It is observed that for parallel flows with $\eta = 1.0$ and 10.0, the ratio remains about 0.6313. But for counter-flows with $\eta = -1.0$ and -10.0, the ratio of the Nusselt numbers reach infinitely high values for particular core sizes around $\omega = 0.4745$ and 0.3348, respectively. It must be noted that the precise variation of the ratios for ω values near the asymptotes is more complicated than that shown in the figure. The variation of the ratio around these ω values shown in the figure represents a simplified variation.

In Fig. 2, a comparison of the Nusselt numbers without heat generation to that with heat generation is presented by plotting the ratio of (Nu_i) without heat generation to (Nu_i) with heat generation, against the dimensionless radius of the separating wall. In the limiting case of $\omega = 1$, which corresponds to a simple pipe flow, the ratio is 0.6093. For parallel flow arrangements with $\eta = 1.0$ and 10.0, the ratio variation has two asymptotes. In the case of $\eta = 1.0$, except in the region between $\omega = 0.5290$ and 0.5770, the ratio is positive. The physical significance of the negative values for the ratio is due to the fact that heat generation reverses the direction of the heat flux at the inner separating surface. In the case of $\eta = 10.0$ similar behavior holds. For counter-flow arrangements the variation of the ratio is mainly reversed. The special points of these variations are also shown in the figure.

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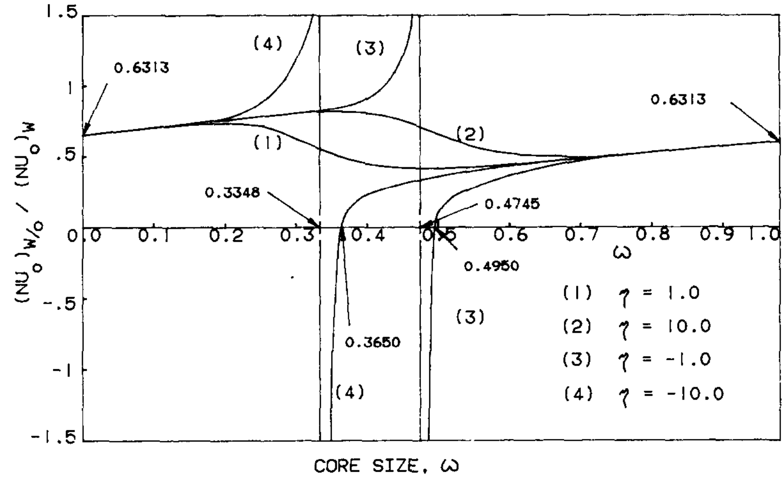

FIG. 1. Ratio of outer Nusselt number without heat generation to with heat generation.

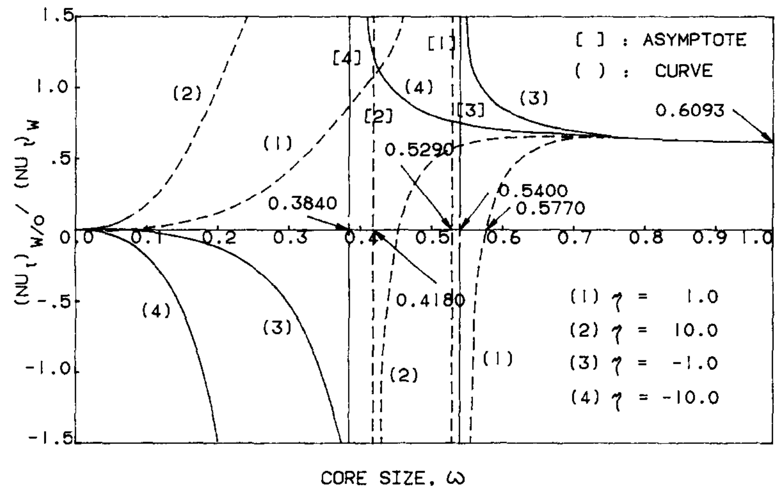

FIG. 2. Ratio of inner Nusselt number without heat generation to with heat generation.

REFERENCES

1. R. K. Shah and A. L. London, *Laminar Flow Forced Convection in Ducts*. Academic Press, New York (1978).
2. R. I. Soloukhin and O. G. Martynenko, Heat and mass transfer bibliography—Soviet literature, *Int. J. Heat Mass Transfer* **26,** 1771–1781 (1983).
3. E. R. G. Eckert, R. J. Goldstein, E. Pfender, W. E. Ibele, S. V. Patankar, J. W. Ramsey, T. W. Simon and N. A. Deckor, Heat transfer—a review of the 1982 literature, *Int. J. Heat Mass Transfer* **26,** 1733–1770 (1983).
4. R. E. Lundberg, W. C. Reynolds and W. M. Kays, Heat transfer with laminar flow in concentric annuli with constant and variable wall temperature with heat flux, NASA Technical Note, TN D-1972, Washington, D.C. (1963).
5. W. M. Kays and M. E. Crawford, *Convective Heat and Mass Transfer*. McGraw-Hill, New York (1980).
6. E. R. G. Eckert and R. M. Drake, *Analysis of Heat and Mass Transfer*. McGraw-Hill, New York (1972).
7. B. Gebhart, *Heat Transfer*. McGraw-Hill, New York (1971).
8. L. C. Burmeister, *Convective Heat Transfer*. Wiley, New York (1983).
9. V. S. Arpaci and P. S. Larsen, *Convective Heat Transfer*. Prentice-Hall, New York (1984).
10. A. Bejan, *Convective Heat Transfer*. Wiley-Interscience, New York (1984).
11. M. A. Ebadian, H. C. Topakoglu and O. A. Arnas, The effect of heat generation on convective heat transfer for laminar flows in a multi-passage circular pipe subjected to an external uniform heat flux, AIAA/ASME Thermophysics and Heat Transfer Conference, ASME Paper 86-HT-4 (1986).
12. M. A. Ebadian, H. C. Topakoglu and O. A. Arnas, Convective heat transfer for laminar flows in a multi-passage circular pipe subjected to an external uniform heat flux, *Int. J. Heat Mass Transfer* **29,** 107–117 (1986).
13. M. A. Ebadian, H. C. Topakoglu and O. A. Arnas, The effect of heat generation on the convective heat transfer within a tube having a constant wall temperature and an elliptical cross section, ASME Paper 85-WA/HT-57 (1985).
14. H. C. Topakoglu and O. A. Arnas, Convective heat transfer for steady laminar flow between two confocal elliptic pipes with longitudinal uniform wall temperature gradient, *Int. J. Heat Mass Transfer* **17,** 1987–1998 (1974).
15. J. J. Duderstadt and L. J. Hamilton, *Nuclear Reactor Analysis*. Wiley, New York (1976).